The witch of Agnesi
Exploitation d'un document audio

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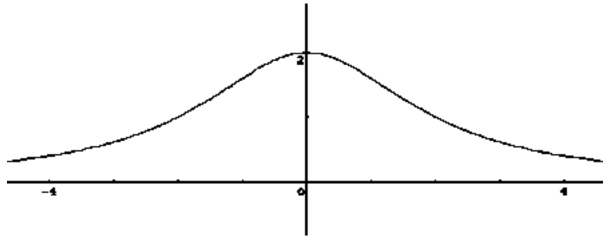
Dans le cadre de la DNL (discipline non linguistique), enseignement d'une matière en langue étrangère pour les élèves de sections européennes, il est fréquent d'utiliser un document audio ou une vidéo pour introduire un thème et faire acquérir du vocabulaire, mathématique ou non. Je présente dans cet article une séquence d'environ deux heures ayant pour point de départ un document audio, autour de Maria Agnesi et de sa célèbre courbe surnommée la sorcière. Cette courbe a pour équation $y = \frac{8}{4 + x^2}$ et pour représentation :



Cette séquence est adaptée aux élèves de TS.

1. Vocabulary loop

Il existe de nombreux moyens pour préparer l'écoute d'un document audio en aidant à comprendre les mots et expressions qui risquent de poser problème aux élèves. L'un d'eux est la boucle de vocabulaire. Le principe en est simple : on prépare une grille dans laquelle se trouvent dans la colonne de gauche des mots ou expressions et dans la colonne de droite, en correspondance, une explication, une définition, ou quelque chose en lien direct avec le mot ou l'expression. Dans la suite, j'utiliserai le mot correspondance, car le mot définition est trop précis dans certains cas. Ceci sera éclairci en observant la grille ci-dessous.

Puis on décale d'une ligne la colonne de droite (facile avec un tableur !), et on découpe chaque ligne. Chaque élève reçoit une ligne, qui contient donc un mot ou une expression, et une correspondance pour un autre mot ou une autre expression.

Un élève lit la partie de droite, et l'élève qui pense avoir le mot ou l'expression correspondant lève la main, et lit ensuite sa propre partie de droite. Le jeu se termine lorsque l'élève qui a commencé termine, d'où le nom de boucle.

Ce jeu fonctionne aussi très bien pour travailler la lecture du vocabulaire mathématique, nombres, fractions, etc., et dans ce cas on a par exemple à gauche la fraction $\frac{3}{4}$, et à droite la façon de la lire, three fourth.

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À l'issue de ce jeu, on donne aux élèves le tableau remis dans l'ordre.

Tableau dans l'ordre :

A witch	A woman practising sorcery
Analytical geometry	A part of geometry dealing with coordinates
A curve	The graph of a function
Maria Agnesi	Name of a famous female mathematician
Differential calculus	A part of mathematics dealing with derivatives
Integral calculus	A part of mathematics dealing with integration
A treatise	A book dealing with a specific subject, often a scientific one
A solution	What you are expected to find when dealing with an equation
Retiring	Something you would say about someone who is very discret, shy, who doesn't mix easily with other people
A prodigy	A extraordinarily gifted person
The apple of his eye	who is very important for him
To strike a deal	To make an arrangement between two persons, with advantages for both
A physicist	A person working in the field of physics
An appointment	A job, but also a meeting

Tableau décalé :

A witch	What you are supposed to do when you have a problem
Analytical geometry	A woman practising sorcery
A curve	A part of geometry dealing with coordinates
Maria Agnesi	The graph of a function
Differential calculus	Name of a famous female mathematician
Integral calculus	A part of mathematics dealing with derivatives
A treatise	A part of mathematics dealing with integration
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A prodigy	Something you would say about someone who is very discret, shy, who doesn't mix easily with other people
The apple of his eye	A extraordinarily gifted person
To strike a deal	who is very important for him
A physicist	To make an arrangement between two persons, with advantages for both
An appointment	A person working in the field of physics
To solve	A job, but also a meeting

2. Présentation du site http://www.uh.edu/engines/

Ce site, nommé *The Engines of our ingenuity*, est un recueil d'émissions de radio qui ont été programmées depuis 1988 sur Public Radio. Le présentateur, John Lienhard, est membre de l'Université de Houston (Texas) et travaille au College of Engineering. Le but du programme est de raconter comment notre culture est formée par la créativité humaine. Les transcriptions tiennent en général sur une page, et on a également accès au document audio. Une grande part des documents traite de sciences, et un bon nombre de mathématiques. Outre le texte lui-même, il est souvent fait référence à des prolongements possibles, comme c'est le cas pour *The Witch of Agnesi*.
Si le prolongement de ce texte n'est adapté que pour les élèves de TS, on trouve sur le site des textes utilisables à tout niveau.

3. Écoute du document

On écoute le document une fois en entier, puis par paragraphes, avec un questionnaire.
Le document se trouve à l'adresse : http://www.uh.edu/engines/epi1741.htm

Quelques minutes sont laissées aux élèves pour lire les questions et vérifier qu'elles sont bien comprises.

Questionnaire

Paragraphs 1 and 2 :

- What is the subject of this episode ?
- What is John Lienhard's book about ?
- What is the question raised by the name of the curve ?

Paragraph 3

- When did Maria Agnesi live and where ?
- When and where was she born ?
- Was she an ordinary child ?
- What is the expression used to show that her father loved her very much ?

Paragraph 4

- What is said about Maria's personality ?
- What happened when she was twenty ? What was the consequence ?
- What did she want to become?
- What was the deal she struck with her father?

Paragraph 5

- What was her first book about ?
- What did an observer say about the open meeting in which she defended her book ?

Paragraph 6

- What was her major contribution to mathematics ?
- How old was she when she finished it ?

Paragraph 7

- Who rewarded her for her work ?
- Who arranged for an appointment at the University, and which University ?
- Did she teach there ?
- What did she do during 45 years ?

Paragraph 8

- How old was Maria when her father died ?
- What did she do then ?
- How old was she when she died ?

Paragraph 9 and 10

- Why is it strange that her curve was named the witch of Agnesi ?
- What is the explanation for this strange name ?
- Who made the mistake ?

4. Étude du texte

Le texte est ensuite distribué et la compréhension vérifiée.

THE WITCH OF AGNESI
by John H. Lienhard

Today, we meet the Witch of Agnesi. The University of Houston's College of Engineering presents this series about the machines that make our civilization run, and the people whose ingenuity created them.

I have this neat old book on analytical geometry. The solution to one of its problems is a singularly gentle and graceful curve. It's called *The Witch of Agnesi*. But why? There's nothing sinister about this flowing line.

It turns out that one person who worked with this curve was a noted eighteenth-century Italian mathematician named Maria Agnesi. So, problem solved : she was the Witch. Well, wait a minute. Let's meet the woman herself. Maria Agnesi was born in

Milan in 1718. Her father was a gentleman intellectual. She was a child prodigy and the apple of his eye.


Maria Agnesi 1718-1799

Yet Maria was shy and retiring, and, when she was twenty, her mother died, leaving the household in her care. She wanted to become a nun, but her father wouldn't hear of that. So they struck a deal. She would stay at home if she could attend church whenever she chose, dress simply, and avoid secular socializing.

In that same year she published her first book — a set of 191 well-organized philosophical propositions, what we would call a physics text. She defended it (as one might a thesis) in an open meeting. One observer said, “ She spoke like an angel. ”

Her major contribution was a two-volume treatise on the new differential and integral calculus, which she finished at the age of thirty. It was a major step in organizing the calculus and bringing it into general use. In this extremely important work, she showed how the calculus could be used to create the curve that would later be called *The Witch of Agnesi*.

Her queen rewarded the treatise with a gift, and the Pope arranged for her appointment to the faculty of the University of Bologna. She remained on their rolls for forty-five years, although she never took up the appointment. Instead, she did charitable works and taught mathematics in one wing of her father's house.

When Agnesi was 34, her father died. Then she put mathematics aside and got on with her real business. She turned to charitable work among the poor and sick. She sold her belongings to raise money to create a retirement home for the poor. She worked steadily until she herself died in that poorhouse at the age of 81. After that, our “ witch ” was labeled “ an *angel* of consolation ”.

Here was a woman who seized the world's attention with brilliant mathematical work and then gave it all up for a half-century of self-sacrifice. That is not the stuff we usually take witches to be made of. So where did the term *Witch of Agnesi* come from?

Well, that curve was called a *versiera*, or *that-which-turns*. But the Italians have another word, *l'aversiera*, which means *she-devil* or *witch*. When a Cambridge professor translated her book into English, he turned *versiera* into *witch* — into *l'aversiera*.

And that is how this saintly mathematician became a witch.

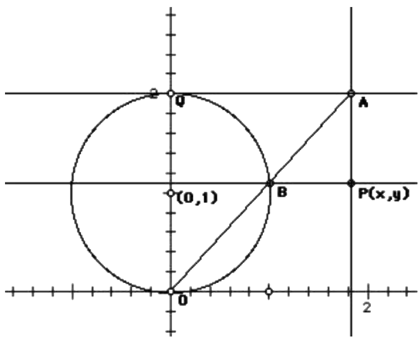
5. Étude de la courbe.

On utilise Géoplan, ou un autre logiciel de géométrie dynamique. Avec Géoplan on a la trace, ce qui est agréable.

Un élève a le texte, et lit les instructions à un élève qui utilise le logiciel, et tous les élèves contrôlent qu'il respecte bien les consignes.
Le texte ci-dessous est distribué aux élèves, sauf à celui qui construit (dans un premier temps).

WITCH CONSTRUCTION

We begin with a circle of radius 1 centred at (0,1). To construct this curve, we first construct the aforementioned circle. The line $y = 2$, which is tangent to the circle at point Q (0,2) is constructed and a point anywhere on that line is chosen, call it point A. A segment is drawn from point A to the bottom point, O (0,0), of the circle. Where this segment OA intersects the circle we construct the point B. Then a line, l, parallel to $y = 2$ and through B is drawn and a line, m, perpendicular to l is constructed through point A. The intersection of l and m is point P. This is the point whose locus is traced, as A moves along $y = 2$ creating the witch curve.



Équation of the curve (voir l'annexe pour les réponses)

From this construction we can generate an equation for the witch curve. We do this by expressing that point B belongs to the circle. The following equalities, and the figure above, help us in this generation of the following equations:

a) As P has x for abscissa and y for ordinate, what is the ordinate of B ?
b) What is the equation of the circle ?
c) What is then the relation between the abscissa X_B of B and its ordinate ?
d) Using Thales' theorem, write another relation between X_B and y.
e) Using d), express X_B in terms of x and y.
f) Using c), express finally y in terms of x.
g) What is the limit of y when x tends to infinity ?
h) What is the value of y if $x = 0$?

Conclusion

Le site évoqué ci-dessus m'a aidé dans la préparation de nombreuses séquences et peut également être utilisé par des enseignants d'autres matières.
Les élèves sont tous sollicités dans des activités comme la vocabulary loop, de même que lorsqu'il s'agit de répondre aux questions.

Lorsque cela est possible, l'écoute du texte et la réponse aux questions peut se faire en labo de langue, ce qui permet à chaque élève d'écouter à son rythme. Les élèves les plus rapides peuvent alors commencer l'étude de la courbe, ou chercher des informations complémentaires sur Maria Agnesi.

Annexe :

Solution of the problem

a) The ordinate of B is also y.
b) The equation of the circle is $x^2 + (y - 1)^2 = 1$.
c) The relation is $X_B{}^2 + (y - 1)^2 = 1$, since B belongs to the circle.
d) Using Thales' theorem in triangle OQA, we have :

$$\frac{x}{x_B} = \frac{2}{y}.$$

e) So $x_B = \dfrac{xy}{2}$.

f) Since $X_B{}^2 + (y - 1)^2 = 1$, we obtain $\dfrac{x^2y^2}{4} + y^2 - 2y + 1 = 1$.

$$\frac{x^2y^2}{4} + y^2 - 2y = 0.$$

X_B cannot be equal equal to 0, because since O, A and B are on the same line, it would mean that x_A is also equal to 0 which is never the case. So y cannot be equal to zero considering the equality obtained in e).
Simplifying by y, and multiplying by 4, we obtain $x^2y + 4y - 8 = 0$, then $y(x^2 + 4) = 8$, and finally

$$y = \frac{8}{4 + x^2}.$$

So the curve equation is $y = \dfrac{8}{4 + x^2}$.

g) If x tends to infinity then y tends to 0.
h) If $x = 0$, $y = 2$, so P is Q.